Python for Development of OpenMP and CUDA Kernels for Multidimensional Data

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Overview

• Use Python environment
  - Problem setup, data structure manipulation, file I/O
  - The “architecture of the computation”

• Implement optimal computation kernels in C++, Fortran, CUDA or 3rd Party APIs
  - Leverage experts and existing code subroutines
  - The “details” of the computation

“Raising the level of programming should be the single most important goal for language designers, as it has the greatest effect on programmer productivity.”

J. Osterhout [14]
Boltzmann Transport Equation

The Boltzmann transport equation for the special case of one dimensional, spherical symmetry, discrete ordinates, time-independent transport is

$$\mu \frac{\partial}{\partial r} \left[ r^2 \psi(r, \mu, E) \right] + \frac{1}{r} \frac{\partial}{\partial \mu} \left[ (1 - \mu^2) \psi(r, \mu, E) \right] + \sigma(r, E) \psi(r, \mu, E) =$$

$$\int_{E}^{1} 2\pi \int_{-1}^{1} \sigma_s(r, \mu, \mu', E \rightarrow E) \psi(r, \mu', E') d\mu' dE' + q(r, \mu, E)$$

Where

- $\psi$ is the radiation intensity (flux) at position $r$, with energy $E$ moving in $\mu$
- $\sigma$ and $\sigma_s$ are the total and scattering cross-sections
- $q$ is the external source particle density

To solve numerically, we discretize in energy, angle and radial terms.
Energy Discretization

- Choose number of energy groups ($G$) and $E_{\text{Max}}$ to correspond to the resolution of interest.
- Energy groups may be of different sizes, depending on resolution of interest.
Angular and Radial Discretization

\[ \mu = \cos\theta \]

Gauss-Legendre Angular quadrature

Toward sphere center

Toward sphere boundary

Diamond Difference Method

Sphere center

Sphere boundary
Angular and Radial Discretization

http://www.oar.noaa.gov/climate/t_modeling.html
“Sweep” radial cells within Each Energy Group

- A transport “sweep” is the process of solving the diamond difference, space-angle $S_N$ equations
  - A wavefront solution in which the value of each cell depends on the flux entering in the “upwind” direction.
Algorithm Structure and Profile

DATA is contiguous in memory

AngluarFluxContainer
SourceContainer
Mesh, misc vectors

GPU Memory

Solve()

35% in misc.
routines

Internal loop
over number
of cells

Z*M*G*L iterations

# of loops varies

G

μ

For all energy groups

Unit convergence

For all angles

19%

7.5%

7.5%

1%

1%
def prob1(Z,M,G,L,a_sxs,a_ofm,a_ext,a_mu):
    r_src = zeros([G,Z,M]).astype(a_ext.dtype)
    for z in range(0,Z):
        for m in range(0,M):
            ss = 0.0
            for g in reversed(range(0,G)):
                for el in range(0,L+1):
                    # NB: [0,L+1)
                    v = plgnr(el,a_mu[m])
                    ss = ss + (2*el+1)/(4*(PI)) * a_sxs[G-1,g,el] * v * a_ofm[g,z,el]
                    r_src[G-1,z,m] = ss + a_ext[G-1,0]/(4*(PI))
            return r_src
namespace ce
{
    template<typename T>
    void prob1(const int Z, 
                const int M, 
                const int G, 
                const int L, 
                const T *a_sxs, // read-only: a_szx[G][G][L+1] 
                const T *a_ofm, // read-only: a_ofm[G][Z][L+1] 
                const T *a_ext, // read-only: a_ext[G] 
                const T *a_mu, // read-only: a_mu[M] 
                T *r_src) // write-only: r_src[G][Z][M]
    {
        const T PI = 3.1415926535827; 
        T ss, v; 
        int z,m,g,l; 
        uint32_t i_src,i_sxs,i_ofm; 
        #pragma omp parallel for shared(a_sxs,a_ofm,a_ext,a_mu,r_src) \ 
        private(ss,v,z,m,i_src,g,l,i_sxs,i_ofm)
        for(z = 0; z < Z; ++z) { 
            for(m = 0; m < M; ++m) { 
                ss = 0.0; 
                i_src = (G-1)*(Z*M) + z*M + m; 
                // r_src[G-1][z][m] 
                for(g = G-1; g >= 0; --g) { 
                    for(el = 0; el <= L; ++el) { 
                        // NB: [0,L+1] 
                        v = gsl::plqndr<T>(el, a_mu[m]); 
                        i_sxs = (G-1)*(G*(L+1)) + g*(L+1) + el; 
                        // a_sxs[G-1][g][el] 
                        i_ofm = g*(Z*(L+1)) + z*(L+1) + el; 
                        // a_ofm[g][z][el] 
                        ss += (2.0*el + 1.0) / (4.0*PI) * a_sxs[i_sxs] * v * a_ofm[i_ofm]; 
                    } 
                    r_src[i_src] = ss + a_ext[G-1] / (4.0*PI); 
                } 
            } 
        }
    }
}
Flow for C++ Wrapper

1. **setup_c.py**
2. **xxx_c.pyf**
3. **xxx_c_wrap.c**
4. **xxx_c.cxx**
5. **xxx_c.h**

**g++**

**libxxx_c.a**

**python**

**xxx_c.so**

Dynamic library containing subroutine and wrapper, ready to import into python scripts

**xxx.py**

Test script imports functions from ‘xxx.py’ to call subroutine defined in ‘xxx_c.h’
python module _prob1_c
interface
  subroutine prob1_dp(z,m,g,l,sxs,ofm,ext,mu,src)
    intent(c) prob1_dp             ! is a C function
    intent(c)                     ! all arguments are
    ! considered as C based
    integer intent(in) :: z
    integer intent(in) :: m
    integer intent(in) :: g
    integer intent(in) :: l
    real*8  intent(in),dimension(g,g,l+1),depend(g,l) :: sxs
    real*8  intent(in),dimension(g,z,l+1),depend(g,z,l) :: ofm
    real*8  intent(in),dimension(g),depend(g) :: ext
    real*8  intent(in),dimension(m),depend(m) :: mu
    real*8  intent(out),dimension(g,z,m),depend(g,z,m) :: src
  end subroutine prob1_dp
Python C++/F2PY Interface Building
(setup_c.py and makefile)

# File setup_c.py

def configuration(parent_package='', top_path=None):
    from numpy.distutils.misc_util import Configuration
    config = Configuration('', parent_package, top_path)

    config.add_library(name='prob1_c', sources=['prob1_c.cxx'])
    config.add_extension('_prob1_c',
        sources = ['prob1_c.pyf', 'prob1_c_wrap.c'],
        libraries = ['prob1_c'])

    return config

if __name__ == '__main__':
    from numpy.distutils.core import setup
    setup(**configuration(top_path='').todict())

# build OpenMP-versions
omp:
@(
    export ARCHFLAGS=$\{ARCHFLAGS\} ; \n
    export CPPFLAGS="-fopenmp $\{TUNE\}" ; \n
    export LDFLAGS="-lgomp" ; \n
    python setup_c.py build_src build_ext --inplace )
Flow for C++ Wrapper (again)
# interface C-code via F2PY

def prob1_c_f2py(Z, M, G, L, a_sxs, a_ofm, a_ext, a_mu):
    import _prob1_c as c_f2py
    if len(Z.shape) > 1:
        Z = Z[0, 0]
    if len(M.shape) > 1:
        M = M[0, 0]
    if len(G.shape) > 1:
        G = G[0, 0]
    if len(L.shape) > 1:
        L = L[0, 0]
    r_src = zeros([G, Z, M]).astype(a_ext.dtype)
    if a_ext.dtype == "float64":
        r_src = c_f2py.prob1_dp(Z, M, G, L, a_sxs, a_ofm, a_ext, a_mu)
    else:
        r_src = c_f2py.prob1_sp(Z, M, G, L, a_sxs, a_ofm, a_ext, a_mu)
    return r_src
Flow for CUDA Wrapper

- **setup_g.py**
- **xxx_g.pyf**
- **xxx_g.cxx**
- **xxx_g.h**
- **xxx_kernel.cu**

**Static library containing C-callable interface “presentation”**

**libxxx_g.a**

**g++**

- **libxxx_g.a**
- **xxx_kernel.o**

**nvcc**

**Dynamic library containing subroutine and wrapper, ready to import into python scripts**

**xxx.g.so**

**python**

- **test.py**
- **python**

**xxx.py**

**Test script imports functions from ‘xxx.py’ to call subroutines defined in ‘xxx_kernel.cu’**
def configuration(parent_package='', top_path=None):
    from numpy.distutils.misc_util import Configuration
    config = Configuration('', parent_package, top_path)
    config.add_library(name='prob1_g',
                       sources=['prob1_g.cxx'])
    config.add_extension('_prob1_g',
                         sources=['prob1_g.pyf', 'prob1_c_wrap.c'],
                         extra_objects=['prob1_kernel.o'],
                         libraries=['prob1_g', 'cuda', 'cudart'])
    return config

if __name__ == '__main__':
    from numpy.distutils.core import setup
    setup(**configuration(top_path='').todict())
# build GPU-versions
$(MOD)_kernel.o: $(MOD)_kernel.h $(MOD)_kernel.cu
   nvcc $(NVCC_CU_FLAGS) $(INCLUDES) -c $(MOD)_kernel.cu

_$(MOD)_g.so: $(MOD)_kernel.o $(MOD)_g.h $(MOD)_g.cxx $(MOD)_c_wrap.c $(MOD)_g.pyf setup_g.py
   ( export ARCHFLAGS=$(ARCHFLAGS) ; \
     export CPPFLAGS="-fopenmp $(TUNE)" ; \
     export LDFLAGS="-L$(CLIB) -lgomp" ; \
     python setup_g.py build_ext --inplace )

gpu: _$(MOD)_g.so
“problem set #1”

- Loop over all radial cells (Z)
- Loop over all angles (M, typically 8)
- Loop over all energy groups (G)
- Integrate Legendre expansion for this angular moment, accumulate the source term (L ~ 24th order)
- Update the radial cell source term

1. Compute Legendre table in CPU memory
2. Copy Legendre table lookup to constant memory on GPU
3. Copy solver state to GPU (Unit Test Only)
4. Load Shared Memory
5. Execute
6. SP’s work
7. Copy result (angular flux) to CPU memory (Unit Test Only)
Runtime Comparison (with I/O overhead)

- Verified matching results for single-precision, double-precision ~ 1e-6
- Fermi implements accelerator-model speedup of 1.3x to 6.2x
  - accounting for the I/O to and from CPU memory
  - versus 2 and 4 core CPUs
Runtime Comparison (kernel only)

- NOTE: logarithmic scale
- Kernel-only timing shows 65x to 115x speedup (vs. 2-core CPU)
  - OK, because our final code has all data resident on the GPU memory
- Significant performance differences between experimental systems

```plaintext
<table>
<thead>
<tr>
<th>G</th>
<th>Z</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1000</td>
<td>0.001</td>
</tr>
<tr>
<td>100</td>
<td>4000</td>
<td>0.01</td>
</tr>
<tr>
<td>500</td>
<td>40000</td>
<td>0.1</td>
</tr>
</tbody>
</table>
```

“prob1”, 24th order Legendre expansion, 8 angles
Performance Model

\[ LOAD = (Z \times M \times L)(G^2 + G), \]
\[ FLOPS = (Z \times M) \times \left( \frac{11}{2} (L^2 + L)(G^2 + G) + 2 \right), \]
\[ STORE = (Z \times M). \]

\[ T_{Estimated} = \frac{(LOAD + STORE) \times P_{bits} \times P_{mem}}{M_{clk} \times M_{width}} \]
\[ + \frac{FLOPS \times P_{fpu}}{P_{clk} \times P_{num}} \]

- \( P_{mem} \) and \( P_{fpu} \) are efficiency factors applied (simplified model)
- \( P_{bits} \) is 64 (IEEE-754 double-precision)
- Applied to both CPU and GPU (naïve)
CPU/GPU Comparison (with I/O overhead)

- Measured vs. Estimated Runtime (performance model)
  - M2070 factors in a 2.5 second application load delay (CUDA overhead)
- M2070 (448 cores, 225W) similar to Dual X5670 (12 cores, 190W)
  - M2070 \{P_{\text{mem}}=46, P_{\text{fpu}}=50 (2\%)\}
  - X5670 \{P_{\text{mem}}=12, P_{\text{fpu}}=58 (1.7\%)\}
CPU/GPU Comparison (with I/O overhead)

- Measured vs. Ideal Runtime (performance model)
  - $P_{\text{mem}}$ and $P_{\text{fpu}}$ set to 1
- M2070 (448 cores, 225W) similar to Dual X5670 (12 cores, 190W)
  - Keeping in mind that we are factoring the I/O overhead
M2070 “Fermi” GPU
Multi-Core CPU, GPU, FPGA
“Exploratory System”
Next Task (#2) has Loop Dependency

DATA is contiguos in memory

35% in misc. routines

Internal loop over number of cells

Z*M*G*L iterations

# of loops varies

19%

19%

19%

19%
Computational Engine: multi-core CPU with GPU and FPGA
Summary

• Use Python environment
  - Problem setup, data structure manipulation, file I/O
  - Use the wide array of available modules
  - Syntax similar to Matlab (the scientists will like it)

• Implement optimal computation kernels in C++, Fortran, CUDA or 3rd Party APIs
  - Leverage experts and existing code subroutines
  - Opportunities to use ASIC/Heterogenous Computation Devices (via API calls)

• All code referenced in this paper
  - [link](http://info.ornl.gov/sites/publications/Files/Pub30033.tgz)

“Raising the level of programming should be the single most important goal for language designers, as it has the greatest effect on programmer productivity.”

J. Osterhout [14]
Thank You